

Remarks: (answer the following questions... assume any missing data)

**Problem number (1) (16 Marks)**

- (a) Check whether the following systems are static, linear, shift invariant, causal, and stable. Explain your answer. (10 Marks)

(i)  $y_1(n) = x^2(n)$

(ii)  $y_2(n) = \cos(x(n))$

- (b) Consider the discrete-time sequence  $x(n]$  which is periodic every 6 samples: (6 Marks)

$$x((n))_6 = \{2, -1, 0, 1, 3, -2\}$$

Find the circular shift:

(i)  $x((n-2))_6$

(ii)  $x((n+2))_6$

(iii)  $x((-n))_6$

(iv)  $x((2-n))_6$

**Problem number (2) (14 Marks)**

- (a) Find the inverse Z-Transform of the following functions: (8 Marks)

(i)  $X(z) = \frac{z^2 + 0.5z + 1.5}{(z^2 - 1.5z + 0.5)}$

(ii)  $X(z) = \frac{z(z+0.5)}{(z-0.1)^2(z-0.6)}$

- (b) Find Z-Transform and ROC for the following sequences: (6 Marks)

(i)  $x_1(n) = n(0.5)^n$

(ii)  $x_2(n) = (0.2)^n e^n$

**Problem number (3) (16 Marks)**

- (a) Compute the linear convolution,  $y(n) = x(n) * h(n)$ , where (6 Marks)

$$x(n) = \{1, -1, 2, 2\}, \quad h(n) = u(n) - u(n-3)$$

- (b) Determine the 4-point DFT of the following sequence: (6 Marks)

$$x(n) = \{-1, 2, 2, -1\}$$

Sketch the magnitude and phase of the result 4-point DFT



Find the IDFT of  $X(k)$  :

(4 Marks)

$$X(k) = \{6, -2 + j2, -2, -2 - j2\}$$

**Problem number (4) (24 Marks)**

(a) A difference equation describing a digital system is given by:

$$y(n) + 0.5y(n-1) = 2(0.8)^n u(n)$$

(i) Solve the difference equation to find  $y(n)$ .

(3 Marks)

(ii) Find the initial and final values of the system response  $y(n)$ .

(2 Marks)

(iii) Check the system stability.

(2 Marks)

(b) State the difference between the FIR filter and IIR filter.

(2 Marks)

(c) Consider the filter transfer function,

(6 Marks)

$$H(z) = \frac{(1 + 2Z^{-1} + Z^{-2})}{(1 + Z^{-1})(1 - 0.5Z^{-1})(1 + 2Z^{-1})}$$

**Draw**

(i) Direct form I

(ii) Direct form II

(iii) Parallel form

(d) Design a second order digital high pass filter with cutoff frequency of  $2\pi$  rad/sec and sampling rate of 4Hz.

(6 Marks)

(e) Consider the transfer function of an analog filter,

(3 Marks)

$$H(S) = \frac{(S + 0.5)}{(S^2 + 3)}$$

Use bilinear transformation to design the corresponding digital filter ( $T=1$  Sec).

*Good luck*

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$x(n)$	$X(z)$
$\delta(n)$	1
$(a)^n$	$\frac{z}{z-a}$
$u(n)$	$\frac{z}{z-1}$
$e^{-an}$	$\frac{z}{z-e^{-a}}$
$n$	$\frac{z}{(z-1)^2}$